SIMILAR TRIANGLES

6

CONTENTS

➢ CONCEPT OF SIMILARITY

Geometric figures having the same shape but different sizes are known as similar figures. Two congruent figures are always similar but similar figures need not be congruent.

Illustration 1 :

Any two line segments are always similar but they need not be congruent. They are congruent, if their lengths are equal.

Illustration 2 :

Any two circles are similar but not necessarily congruent. They are congruent if their are equal.

Illustration 3 :

(i) Any two square are similar (see fig. (i))

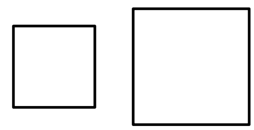


Fig.(i)

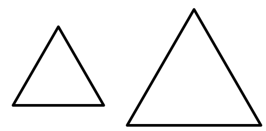


Fig.(ii)

(ii) Any two equilateral triangles are similar (see fig. (ii))

➢ SIMILAR POLYGONS

❖ **Definition**

Two polygons are said to be similar to each other, if

(i) their corresponding angles are equal, and

(ii) the lengths of their corresponding sides are proportional.

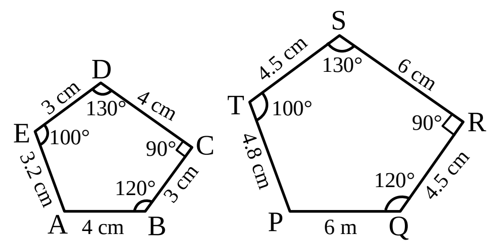
If two polygons ABCDE and PQRST are similar, then from the above definition it follows that :

Angle at A = Angle at P, Angle at B = Angle at Q, Angle at C = Angle at R, Angle at D = Angle at S, Angle at E = Angle at T

and, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DE}{ST}$

If two polygons ABCDE and PQRST, are similar, we write ABCED ~ PQRST.

Here, the symbol '~' stands for is similar to.



SIMILAR TRIANGLE AND THEIR PROPERTIES

Definition

Two triangles are said to be similar, if their

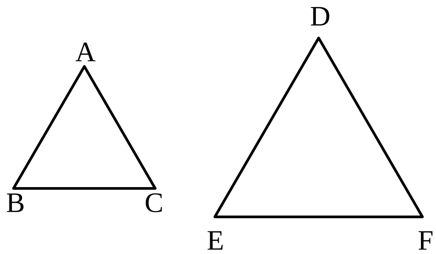
(i) corresponding angles are equal and,

(ii) corresponding sides are proportional.

Two triangles ABC and DEF are similar, if

(i) $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and,

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



SOME BASIC RESULTS ON PROPORTIONALITY

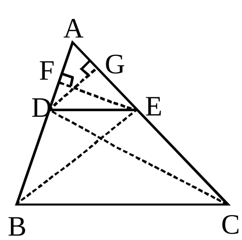
Basic Proportionality Theorem or Thales Theorem

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Given : A triangle ABC in which DE || BC, and intersects AB in D and AC in E.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EF $\perp$ BA and DG $\perp$ CA.



Proof : Since EF is perpendicular to AB. Therefore, EF is the height of triangles ADE and DBE.

Now, Area(ΔADE) $= \frac{1}{2}$ (base × height) $= \frac{1}{2}$ (AD.EF)

and, Area(ΔDBE) $= \frac{1}{2}$ (base × height) $= \frac{1}{2}$ (DB.EF)

$$\therefore \quad \frac{Area(\Delta ADE)}{Area(\Delta DBE)} = \frac{\frac{1}{2}(AD.EF)}{\frac{1}{2}(DB.BF)} = \frac{AD}{DB} \quad .... \text{(i)}$$

Similarly, we have

$$\frac{Area(\Delta ADE)}{Area(\Delta DEC)} = \frac{\frac{1}{2}(AE.DG)}{\frac{1}{2}(EC.DG)} = \frac{AE}{EC} \quad .... \text{(ii)}$$

But, ΔDBE and ΔDEC are on the same base DE and between the same parallels DE and BC.

$\therefore$ Area (ΔDBE) = Area (ΔDEC)

$$\Rightarrow \frac{1}{Area(\Delta DBE)} = \frac{1}{Area(\Delta DEC)}$$

[Taking reciprocals of both sides]

$$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta DBE)} = \frac{Area\ (\Delta ADE)}{Area\ (\Delta DEC)}$$

[Multiplying both sides by Area (ΔADE)]

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[Using (i) and (ii)]}$$

Corollary : If in a ΔABC, a line DE || BC, intersects AB in D and AC in E, then :

(i) $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Proof : (i) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE} \text{ [Taking reciprocals of both sides]}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} \text{ [Adding 1 on both sides]}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{DE}{EC}$$

$$\Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

[Adding 1 on both sides]

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

So, if in a ΔABC, DE || BC, and intersect AB in D and AC in E, then we have

(i) $\frac{AD}{DB} = \frac{AE}{EC}$ (ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$ (iv) $\frac{AD}{AB} = \frac{AE}{AC}$

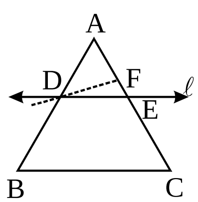
(v) $\frac{AB}{DB} = \frac{AC}{EC}$ (vi) $\frac{DB}{AB} = \frac{EC}{AC}$

❖ Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Given : A ΔABC and a line l intersecting AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : ℓ || BC i.e. DE || BC



Proof : If possible, let DE be not parallel to BC. Then, there must be another line parallel to BC. Let DF || BC.

Since DF || BC. Therefore from Basic Proportionality Theorem, we get

$$\frac{AD}{DB} = \frac{AF}{FC} \quad .... (i)$$

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (Given) .... (ii)

From (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

$\Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$ [Adding 1 on both sides]

$$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

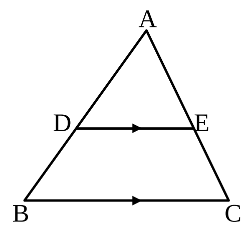
$$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC} \Rightarrow FC = EC$$

This is possible only when F and E coincide i.e. DF is the line l itself. But, DF || BC. Hence, l || BC.

❖ EXAMPLES ❖

Ex.1 D and E are points on the sides AB and AC respectively of a ΔABC such that DE || BC.

Find the value of x, when



(i) AD = 4 cm, DB = (x – 4) cm, AE = 8 cm and EC = (3x – 19) cm

(ii) AD = (7x – 4) cm, AE = (5x – 2) cm,

DB = (3x + 4) cm and EC = 3x cm.

Sol. (i) In ΔABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By thales theorem)

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

$4(3x – 19) = 8(x – 4)$

$12x – 76 = 8x – 32$

$4x = 44$

$x = 17$

(ii) In ΔABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By thales theorem)

$$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$21x^2 – 12x = 15x^2 – 6x + 20x – 8$

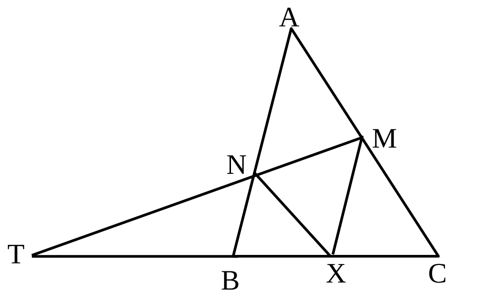
$6x^2 – 26x + 8 = 0$

$3x^2 – 13x + 4 = 0$

$(x – 4)(3x – 1) = 0 \Rightarrow x = 4, 1/3$

Ex.2 Let X be any point on the side BC of a triangle ABC. If XM, XN are drawn parallel to BA and CA meeting CA, BA in M, N respectively; MN meets BC produced in T, prove that $TX^2 = TB \times TC$.

Sol. In ΔTXM, we have



$XM \parallel BN$

$$\therefore \quad \frac{TB}{TX} = \frac{TM}{TN} \qquad \text{.... (i)}$$

In ΔTMC, we have

$XN \parallel CM$

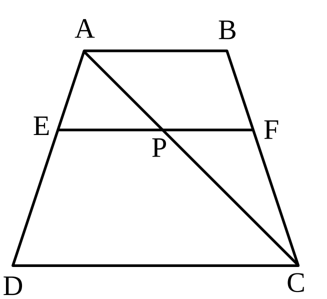
$$\therefore \quad \frac{TX}{TC} = \frac{TN}{TM} \qquad \text{.... (ii)}$$

From equations (i) and (ii), we get

$$\frac{TB}{TX} = \frac{TX}{TC}$$

$\Rightarrow \; TX^2 = TB \times TC$

Ex.3 In fig., $EF \parallel AB \parallel DC$. Prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Sol. We have,

$EF \parallel AB \parallel DC$

$\Rightarrow \; EP \parallel DC$

Thus, in ΔADC, we have

$EP \parallel DC$

Therefore, by basic proportionality theorem, we have

$$\frac{AE}{ED} = \frac{AP}{PC} \qquad \text{.... (i)}$$

Again, $EF \parallel AB \parallel DC$

$\Rightarrow \; FP \parallel AB$

Thus, in ΔCAB, we have

$FP \parallel BA$

Therefore, by basic proportionality theorem, we have

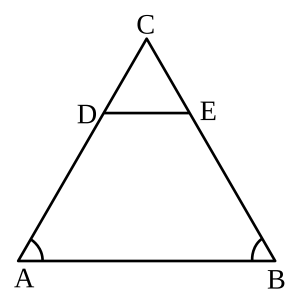
$$\frac{BF}{FC} = \frac{AP}{PC} \qquad \text{....(ii)}$$

From (i) and (ii), we have

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.4 In figure, $\angle A = \angle B$ and $DE \parallel BC$. Prove that $AD = BE$

Sol.



$\angle A = \angle B$ (given)

$\Rightarrow \; BC = AC$ ....(i)

(Sides opposite to equal angles are equal)

Now, $DE \parallel AB$

$$\Rightarrow \; \frac{CD}{DA} = \frac{CE}{EB}$$

(By basic proportionality theorem)

$$\Rightarrow \; \frac{CD}{DA} + 1 = \frac{CE}{EB} + 1$$

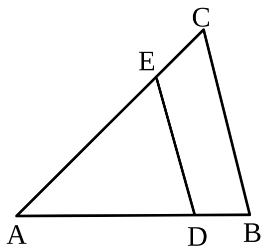
(Adding 1 on both sides)

$$\Rightarrow \; \frac{CD + DA}{DA} = \frac{CE + EB}{EB} \Rightarrow \frac{CA}{DA} = \frac{CE}{EB}$$

$$\Rightarrow \frac{AC}{AD} = \frac{BC}{BE} \Rightarrow \frac{AC}{AD} = \frac{AC}{BE}$$

$$\Rightarrow \frac{1}{AD} = \frac{1}{BE} \Rightarrow AD = BE$$

Ex.5 In fig., DE || BC. If AD = 4x – 3, DB = 3x – 1, AE = 8x – 7 and EC = 5x – 3, find the value of x.



Sol. In ΔABC, we have

DE || BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By Thale's Theorem]}$$

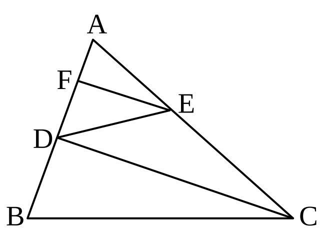
$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (4x - 3)(5x - 3) = (3x - 1)(8x - 7)$$

x = 1

Ex.7 In fig. DE || BC and CD || EF. Prove that $AD^2 = AB \times AF$.

Sol. In ΔABC, we have



DE || BC

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad \text{....(i)}$$

In ΔADC, we have

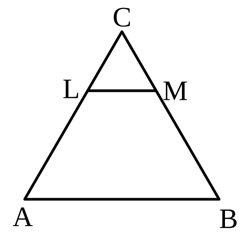
FE || DC

$$\Rightarrow \frac{AD}{AF} = \frac{AC}{AE} \quad \text{....(ii)}$$

From (i) and (ii), we get

$$\frac{AB}{AD} = \frac{AD}{AF} \Rightarrow AD^2 = AB \times AF$$

Ex.9 In fig., LM || AB. If AL = x – 3, AC = 2x, BM = x – 2 and BC = 2x + 3, find the value of x.



Sol. In ΔABC, we have

LM || AB

$$\therefore \frac{AL}{LC} = \frac{MB}{MC} \quad \text{[By Thale's Theorem]}$$

$$\Rightarrow \frac{AL}{AC - AL} = \frac{BM}{BC - BM}$$

$$\Rightarrow \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$$

$$\Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5}$$

$$\Rightarrow (x - 3)(x + 5) = (x - 2)(x + 3)$$

$$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$$

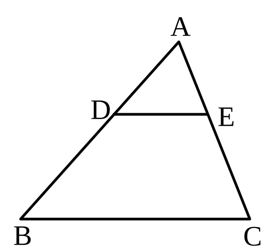
$$\Rightarrow x = 9$$

Ex.10 In a given ΔABC, DE || BC and $\frac{AD}{DB} = \frac{3}{4}$. If AC = 14 cm, find AE.

Sol. In ΔABC, we have

DE || BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By Thales Theorem]}$$



$$\Rightarrow \frac{AD}{DB} = \frac{AE}{AC - AE}$$

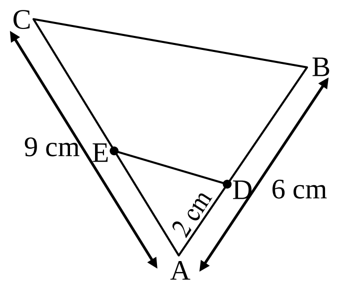
$$\Rightarrow \frac{3}{4} = \frac{AE}{14 - AE} \quad [\because AC = 5.6]$$

$$\Rightarrow 3(14 - AE) = 4AE$$

$$\Rightarrow 42 - 3AE = 4AE$$

$\Rightarrow$ 42 = 7AE $\quad\Rightarrow$ AE = $\frac{42}{7}$ = 6 cm

Ex.11 In figure, DE || BC. Find AE.



Sol. Let AE = x cm

Then EC = (9 – x) cm

AD = 2 cm

DB = (6 – 2) cm = 4 cm

We have $\frac{AE}{BE} = \frac{AD}{DB}$

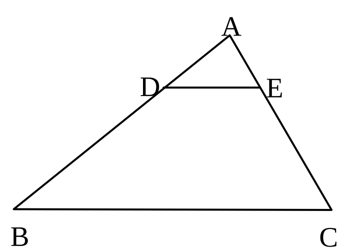
[By Basic Proportionality Theorem]

$\Rightarrow \frac{x}{9-x} = \frac{2}{4}$ $\quad\Rightarrow$ 4x = 2 (9 – x)

$\Rightarrow$ 6x = 18 $\quad\Rightarrow$ x = 3

Hence, AE = 3 cm

Ex.13 In fig., $\frac{AD}{DB} = \frac{1}{3}$ and $\frac{AE}{AC} = \frac{1}{4}$. Using converse of basic proportionality theorem, prove that DE || BC.



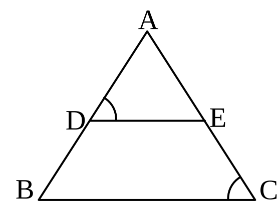
Sol. $\frac{AE}{AC} = \frac{1}{4}$

$\Rightarrow \frac{AC}{AE} = 4 \Rightarrow \frac{AC}{AE} - 1 = 3$

$\Rightarrow \frac{AC - AE}{AE} = 3 \Rightarrow \frac{EC}{AE} = 3 \Rightarrow \frac{AE}{EC} = \frac{1}{3}$

trisection of one side of a triangle parallel to another side trisect the third side.

Ex.15 In the given figure, $\frac{AD}{DB} = \frac{AE}{EC}$ and ∠ADE = ∠ACB. Prove that ΔABC is an isosceles triangle.



Sol. We have,

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow$ DE || BC

[By the converse of Thale's theorem]

∴ ∠ADE = ∠ABC (corresponding ∠s)

But, ∠ADE = ∠ACB (given)

∴ ∠ABC = ∠ACB.

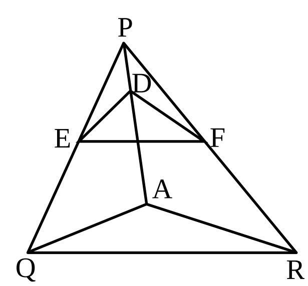
So, AB = AC [sides opposite to equal angles]

Hence, ΔABC is an isosceles triangles.

Ex.16 In fig., if DE || AQ and DF || AR. Prove that EF || QR. **[NCERT]**

Sol. In ΔPQA, we have

DE || AQ **[Given]**



Therefore, by basic proportionality theorem, we have

$\frac{PE}{EQ} = \frac{PD}{DA}$ ....(i)

In ΔPAR, we have

DF || AD [Given]

Therefore, by basic proportionality theorem, we have

$\frac{PD}{DA} = \frac{PF}{FR}$ ....(ii)

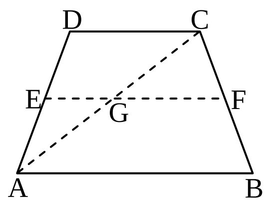
From (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow$ EF || QR [By the converse of Basic Proportionality Theorem]

Ex.18 ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF || AB. Show that

$$\frac{AE}{ED} = \frac{BF}{FC}.$$



Sol. **Given :** A trap. ABCD in which AB || DC. E and F are points on AD and BC respectively such that EF || AB.

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$

Construction : Ioin AC, intersecting EF at G.

Proof : EF || AB and AB || DC

$\Rightarrow$ EF || DC

Now, in ΔADC, EG || DC

$\therefore \frac{AE}{ED} = \frac{AG}{GC}$ ....(i) [By Thale's theorem]

Similarly, in ΔCAB, GF || AB.

$\therefore \frac{AG}{GC} = \frac{BF}{FC}$ ....(ii)

[$\because \frac{GC}{AG} = \frac{FC}{BF}$ by Thale's theorem]

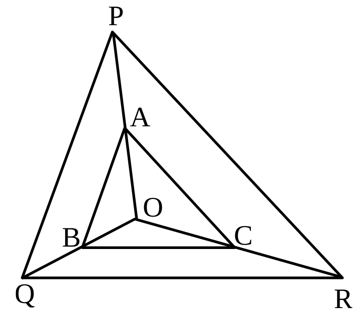
From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.19 In fig., A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR. **[NCERT]**

Sol. In ΔOPQ, we have

AB || PQ



$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ}$ ....(i)

In ΔOQR, we have

BC || QR

$\Rightarrow \frac{OB}{BQ} = \frac{OC}{CR}$ ....(ii)

From (i) and (ii), we get

$$\frac{OA}{AP} = \frac{OC}{CR}$$

Thus, A and C are points on sides OP and OR respectively of ΔOPR, such that

$$\frac{OA}{AP} = \frac{OC}{CR}$$

$\Rightarrow$ AC || PR [Using the converse of BPT]

Ex.20 Any point X inside ΔDEF is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meeting XE at Q and QR is drawn parallel to EF meeting XF in R. Prove that PR || DF.

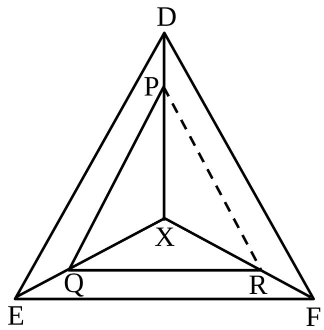
Sol. A ΔDEF and a point X inside it. Point X is joined to the vertices D, E and F. P is any point on DX. PQ || DE and QR || EF.

To Prove : PR || DF

Construction : Join PR.

Proof : In ΔXED, we have

PQ || DE



$\therefore \quad \dfrac{XP}{PD} = \dfrac{XQ}{QE}$ ....(i) [By Thale's Theorem]

In ΔXEF, we have

QR || EF

$\therefore \quad \dfrac{XQ}{QE} = \dfrac{XR}{RF}$ ...(ii) [By Thale's Theorem]

From (i) and (ii), we have

$$\frac{XP}{PD} = \frac{XR}{RF}$$

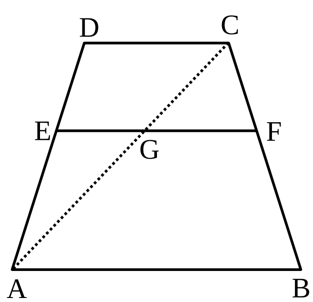
Thus, in ΔXFD, points R and P are dividing sides XF and XD in the same ratio. Therefore, by the converse of Basic Proportionality Theorem, we have, PR || DF

Ex.21 Prove that any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

Sol. **Given :** A trapezium ABCD in which DC || AB and EF is a line parallel to DC and AB.

To Prove : $\dfrac{AE}{ED} = \dfrac{BF}{FC}$

Construction : Join AC, meeting EF in G.



Proof : In ΔADC, we have

EG || DC

$\Rightarrow \quad \dfrac{AE}{ED} = \dfrac{AG}{GC}$ [By Thale's Theorem]....(i)

In ΔABC, we have

GF || AB

$\Rightarrow \quad \dfrac{AG}{GC} = \dfrac{BF}{FC}$ [By Thale's Theorem]....(ii)

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.22 Prove that the line drawn from the mid-point of one side of a triangle parallel of another side bisects the third side.

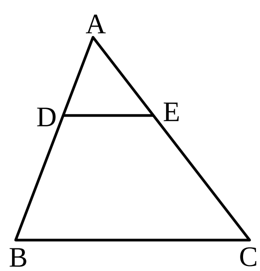
Sol. **Given :** A ΔABC, in which D is the mid-point of side AB and the line DE is drawn parallel to BC, meeting AC in E.

To Prove : E is the mid-point of AC i.e., AE = EC.

Proof : In ΔABC, we have

DE || BC

$\Rightarrow \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$ [By Thale's Theorem] ....(i)



But, D is the mid-point of AB.

$\Rightarrow \quad AD = DB$

$\Rightarrow \quad \dfrac{AD}{DB} = 1$ ....(ii)

From (i) and (ii), we get

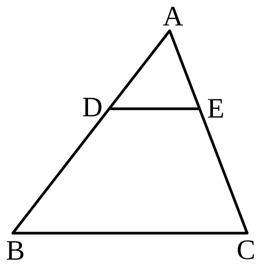
$$\frac{AE}{EC} = 1$$

$\Rightarrow \quad AE = EC$

Hence, E bisects AC.

Ex.23 Prove that the line joining the mid-point of two sides of a triangle is parallel to the third side. **[NCERT]**

Sol. **Given :** A ΔABC in which D and E are mid-point of sides AB and AC respectively.



To Prove : DE || BC

Proof : Since D and E are mid-points of AB and AC respectively.

$\therefore$ AD = DB and AE = EC

$\Rightarrow \dfrac{AE}{DB} = 1$ and $\dfrac{AE}{EC} = 1$

$\Rightarrow \dfrac{AD}{DB} = \dfrac{AE}{EC}$

Thus, the line DE divides the sides AB and AC of ΔABC in the same ratio. Therefore, by the converse of Basic Proportionality Theorem, we have

DE || BC

➤ CRITERIA FOR SIMILARITY OF TRIANGLES

◈ Equiangular Triangles :

Two triangles are said to be equiangular, if their corresponding angles are equal.

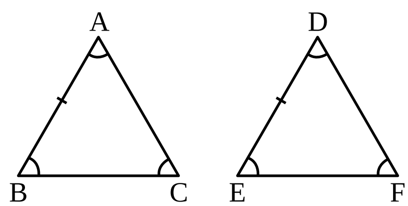
If two triangles are equiangular, then they are similar.

Two triangles ABC and DEF such that

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

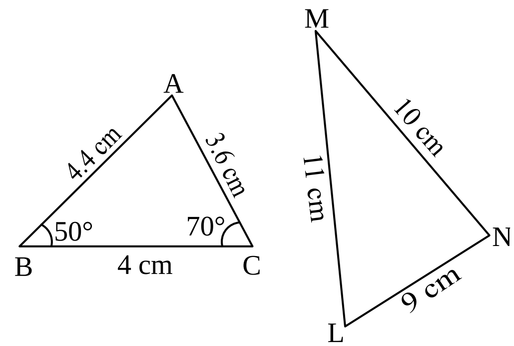
Then $\Delta ABC \sim \Delta DEF$ and

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$



❖ EXAMPLES ❖

Ex.26 In figure, find $\angle L$.



Sol. In ΔABC and ΔLMN,

$\dfrac{AB}{LM} = \dfrac{4.4}{11} = \dfrac{2}{5}$

$\dfrac{BC}{MN} = \dfrac{4}{10} = \dfrac{2}{5}$

and $\dfrac{CA}{NL} = \dfrac{3.6}{9} = \dfrac{2}{5}$

$\Rightarrow \dfrac{AB}{LM} = \dfrac{BC}{MN} = \dfrac{CA}{NL}$

$\Rightarrow \Delta ABC \sim \Delta LMN$ (SSS similarity)

$\Rightarrow \angle L = \angle A$

$= 180^\circ - \angle B - \angle C$

$= 180^\circ - 50^\circ - 70^\circ = 60^\circ$

$\Rightarrow \angle L = 60^\circ$

Ex.27 Examine each pair of triangles in Figure, and state which pair of triangles are similar. Also, state the similarity criterion used by you for answering the question and write the similarity relation in symbolic form.

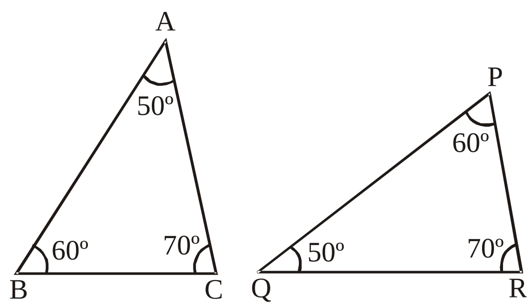


Figure (i)

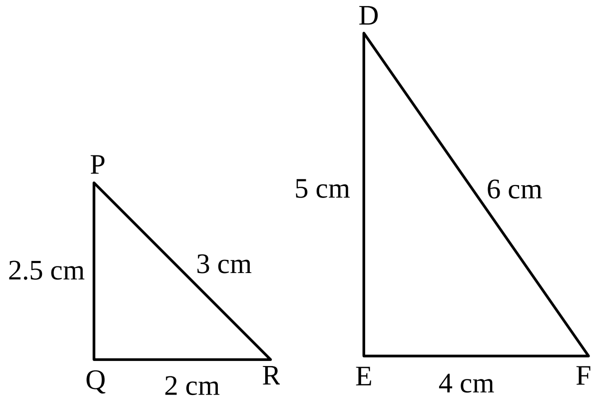


Figure (ii)

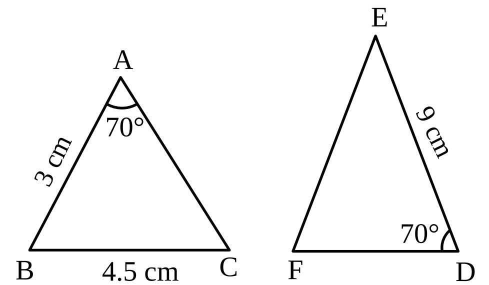


Figure (iii)

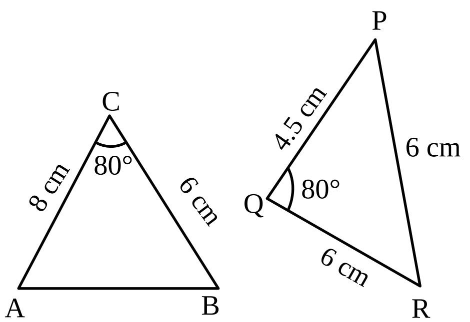


Figure (iv)

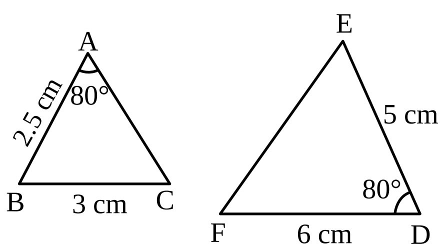


Figure (v)

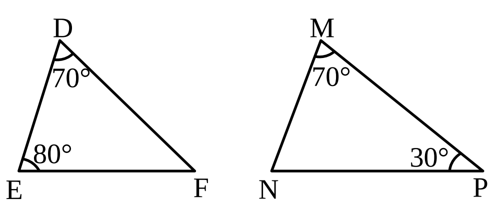


Figure (vi)

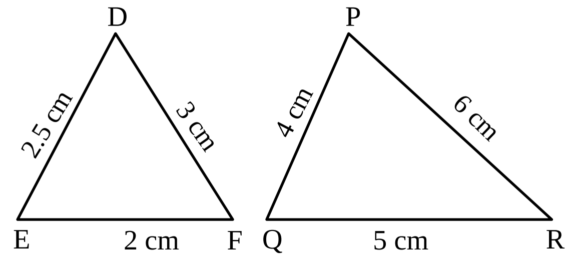


Figure (vii)

Sol. (i) $\angle A = \angle Q$, $\angle B = \angle P$ and $\angle C = \angle R$.

$\therefore$ $\Delta ABC \sim \Delta QPR$ (AAA-similarity)

(ii) In triangle PQR and DEF, we observe that

$$\frac{PQ}{DE} = \frac{QR}{EF} = \frac{PR}{DF} = \frac{1}{2}$$

Therefore, by SSS-criterion of similarity, we have

$\Delta PQR \sim \Delta DEF$

(iii) SAS-similarity is not satisfied as included angles are not equal.

(iv) $\Delta CAB \sim \Delta QRP$ (SAS-similarity), as

$$\frac{CA}{QR} = \frac{CB}{QP} \text{ and } \angle C = \angle Q.$$

(v) In Δ's ABC and DEF, we have

$\angle A = \angle D = 80^\circ$

But, $\dfrac{AB}{DE} \neq \dfrac{AC}{DF}$ [$\because$ AC is not given]

So, by SAS-criterion of similarity these two triangles are not similar.

(vi) In Δ's DEF and MNP, we have

$\angle D = \angle M = 70^\circ$

$\angle E = \angle N = 80^\circ$ [$\because$ $\angle N = 180^\circ - \angle M - \angle P$
$= 180^\circ - 70^\circ - 30^\circ = 80^\circ$]

So, by AA-criterion of similarity

$\Delta DEF \sim \Delta MNP$.

(vii) FE = 2 cm, FD = 3 cm, ED = 2.5 cm

PQ = 4 cm, PR = 6 cm, QR = 5 cm

$\therefore$ $\Delta FED \sim \Delta PQR$ (SSS-similarity)

Ex.28 In figure, QA and PB are perpendicular to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.

Sol. In triangles AOQ and BOP, we have

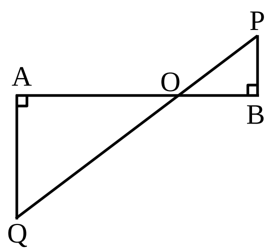
$\angle OAQ = \angle OBP$ [Each equal to 90º]

$\angle AOQ = \angle BOP$

[Vertically opposite angles]

Therefore, by AA-criterion of similarity

$\Delta AOQ \sim \Delta BOP$



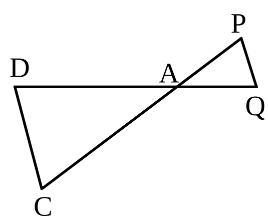
$\Rightarrow \dfrac{AO}{BO} = \dfrac{OQ}{OP} = \dfrac{AO}{BP}$

$\Rightarrow \dfrac{AO}{BO} = \dfrac{AQ}{BP} \Rightarrow \dfrac{10}{6} = \dfrac{AQ}{9}$

$\Rightarrow AQ = \dfrac{10 \times 9}{6} = 15$ cm

Ex.29 In figure, $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ.

Sol. We have, $\Delta ACB \sim \Delta APQ$



$\Rightarrow \dfrac{AC}{AP} = \dfrac{CB}{PQ} = \dfrac{AB}{AQ}$

$\Rightarrow \dfrac{AC}{AP} = \dfrac{CB}{PQ}$ and $\dfrac{CB}{PQ} = \dfrac{AB}{AQ}$

$\Rightarrow \dfrac{AC}{2.8} = \dfrac{8}{4}$ and $\dfrac{8}{4} = \dfrac{6.5}{AQ}$

$\Rightarrow \dfrac{AC}{2.8} = 2$ and $\dfrac{6.5}{AQ} = 2$

$\Rightarrow AC = (2 \times 2.8)$ cm = 5.6 cm and

$AQ = \dfrac{6.5}{2}$ cm = 3.25 cm

Ex.30 The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB.

Sol. Since the ratio of the corresponding sides of similar triangles is same as the ratio of their perimeters.

$\therefore$ $\Delta ABC \sim \Delta PQR$

$\Rightarrow \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = \dfrac{36}{24}$

$\Rightarrow \dfrac{AB}{PQ} = \dfrac{36}{24}$ $\Rightarrow \dfrac{AB}{10} = \dfrac{36}{24}$

$\Rightarrow AB = \dfrac{36 \times 10}{24}$ cm = 15 cm

Ex.31 In figure, $\angle CAB = 90º$ and $AD \perp BC$. If AC = 75 cm, AB = 1 m and BD = 1.25 m, find AD.

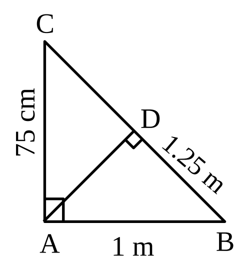
Sol. We have,

AB = 1 m = 100 cm, AC = 75 cm and BD = 125 cm

In ΔBAC and ΔBDA, we have

$\angle BAC = \angle BDA$ [Each equal to 90º]

and, $\angle B = \angle B$



So, by AA-criterion of similarity, we have

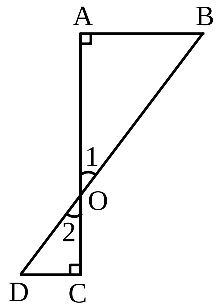
$\Delta BAC \sim \Delta BDA$

$\Rightarrow \dfrac{BA}{BD} = \dfrac{AC}{AD}$

$\Rightarrow \dfrac{100}{125} = \dfrac{75}{AD}$

$\Rightarrow AD = \dfrac{125 \times 75}{100}$ cm = 93.75 cm

Ex.32 In figure, if $\angle A = \angle C$, then prove that $\Delta AOB \sim \Delta COD$.



Sol. In triangles AOB and COD, we have

$\angle A = \angle C$ [Given]

and, $\angle 1 = \angle 2$ [Vertically opposite angles]

Therefore, by AA-criterion of similarity, we have

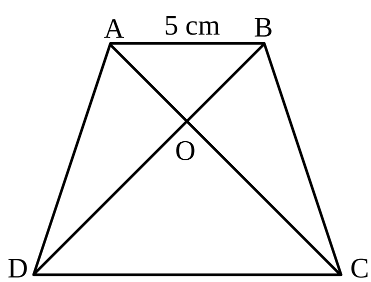
$\Delta AOB \sim \Delta COD$

Ex.33 In figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5 cm. Find the value of DC.

Sol. In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\frac{AO}{OC} = \frac{OB}{OD}$ [Given]



So, by SAS-criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

$\Rightarrow \frac{1}{2} = \frac{5}{DC}$ [$\because$ AB = 5 cm]

$\Rightarrow$ DC = 10 cm

Ex.34 In figure, considering triangles BEP and CPD, prove that $BP \times PD = EP \times PC$.

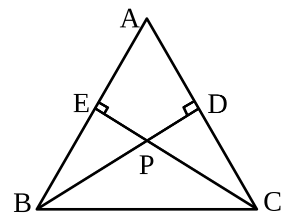
Sol. **Given :** A ΔABC in which $BD \perp AC$ and $CE \perp AB$ and BD and CE intersect at P.

To Prove : $BP \times PD = EP \times PC$

Proof : In ΔEPB and ΔDPC, we have

$\angle PEB = \angle PDC$ [Each equal to 90º]

$\angle EPB = \angle DPC$ [Vertically opposite angles]



Thus, by AA-criterion of similarity, we have

$\Delta EPB \sim \Delta DPC$

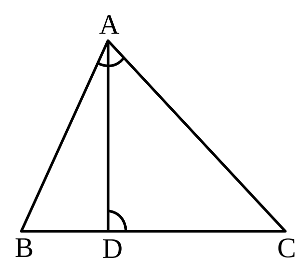
$\frac{EP}{DP} = \frac{PB}{PC}$

$\Rightarrow$ $BP \times PD = EP \times PC$

Ex.35 D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$ or, $CA^2 = CB \times CD$.

Sol. In ΔABC and ΔDAC, we have

$\angle ADC = \angle BAC$ and $\angle C = \angle C$



Therefore, by AA-criterion of similarity, we have

$\Delta ABC \sim \Delta DAC$

$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$

Ex.36 P and Q are points on sides AB and AC respectively of ΔABC. If AP = 3 cm, PB = 6cm. AQ = 5 cm and QC = 10 cm, show that BC = 3PQ.

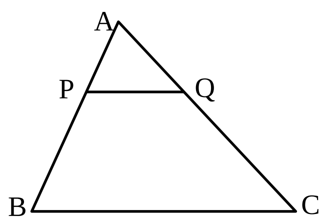
Sol. We have,

$AB = AP + PB = (3 + 6)$ cm $= 9$ cm

and, $AC = AQ + QC = (5 + 10)$ cm $= 15$ cm.

$\therefore \quad \frac{AP}{AB} = \frac{3}{9} = \frac{1}{3}$ and $\frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC}$



Thus, in triangles APQ and ABC, we have

$\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A = \angle A$ [Common]

Therefore, by SAS-criterion of similarity, we have

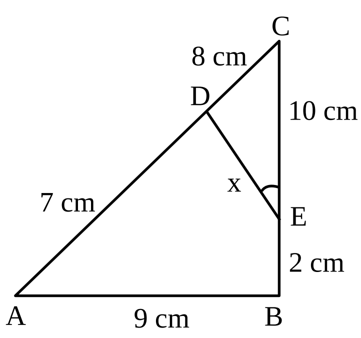
$\Delta APQ \sim \Delta ABC$

$\Rightarrow \quad \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \quad \frac{PQ}{BC} = \frac{AQ}{AC} \Rightarrow \frac{PQ}{BC} = \frac{5}{15}$

$\Rightarrow \quad \frac{PQ}{BC} = \frac{1}{3} \quad \Rightarrow \quad BC = 3PQ$

Ex.37 In figure, $\angle A = \angle CED$, prove that $\Delta CAB \sim \Delta CED$. Also, find the value of x.



Sol. In ΔCAB and ΔCED, we have

$\angle A = \angle CED$ and $\angle C = \angle C$ [common]

$\therefore \quad \Delta CAB \sim \Delta CED$

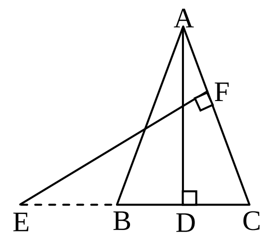
$\Rightarrow \quad \frac{CA}{CE} = \frac{AB}{DE} = \frac{CB}{CD}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{CB}{CD} \Rightarrow \frac{9}{x} = \frac{10+2}{8}$

$\Rightarrow$ x = 6 cm

Ex.38 In the figure, E is a point on side CB produced of an isosceles ΔABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that

$\Delta ABD \sim \Delta ECF$.



Sol. **Given :** A ΔABC in which $AB = AC$ and $AD \perp BC$. Side CB is produced to E and $EF \perp AC$.

To prove : $\Delta ABD \sim \Delta ECF$.

Proof : we known that the angles opposite to equal sides of a triangle are equal.

$\therefore \quad \angle B = \angle C \qquad [\because AB = AC]$

Now, in ΔABD and ΔECF, we have

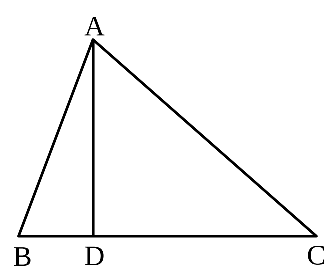
$\angle B = \angle C \qquad$ [proved above]

$\angle ADB = \angle EFC = 90^o$

$\therefore \quad \Delta ABD \sim \Delta ECF$ [By AA-similarity]

Ex.39 In figure, $\angle BAC = 90^o$ and segment $AD \perp BC$. Prove that $AD^2 = BD \times DC$.

Sol. In ΔABD and ΔACD, we have



$\angle ADB = \angle ADC \qquad$ [Each equal to 90^o]

and, $\angle DBA = \angle DAC$

$\begin{bmatrix}\text{Each equal to complement of} \\ \angle BAD \text{ i.e., } 90^\circ - \angle BAD\end{bmatrix}$

Therefore, by AA-criterion of similarity, we have

$\Delta DBA \sim \Delta DAC$

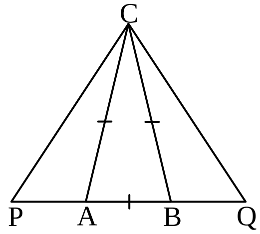
$\begin{bmatrix}\therefore\ \angle D \leftrightarrow \angle D, \angle DBA \leftrightarrow \angle DAC \\ \text{and } \angle BAD \leftrightarrow \angle DCA\end{bmatrix}$

$\Rightarrow \dfrac{DB}{DA} = \dfrac{DA}{DC}$

$\begin{bmatrix}\text{In similar triangles corresponding} \\ \text{sides are proportional}\end{bmatrix}$

$\Rightarrow \quad \dfrac{BD}{AD} = \dfrac{AD}{DC} \quad \Rightarrow \quad AD^2 = BD \times DC$

Ex.40 In an isosceles ΔABC, the base AB is produced both ways in P and Q such that $AP \times BQ = AC^2$ and CE are the altitudes. Prove that $\Delta ACP \sim \Delta BCQ$.



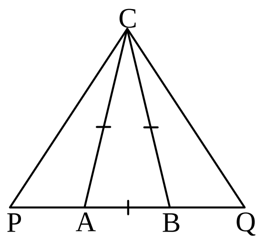
Sol. $CA = CB \Rightarrow \angle CAB = \angle CBA$

$\Rightarrow 180^\circ - \angle CAB = 180^\circ - \angle CBA$

$\Rightarrow \quad \angle CAP = \angle CBQ$

Now, $AP \times BQ = AC^2$

$\Rightarrow \dfrac{AP}{AC} = \dfrac{AC}{BQ} \Rightarrow \dfrac{AP}{AC} = \dfrac{BC}{BQ}$ [$\because$ AC = BC]



Thus, $\angle CAP = \angle CBQ$ and $\dfrac{AP}{AC} = \dfrac{BC}{BQ}$.

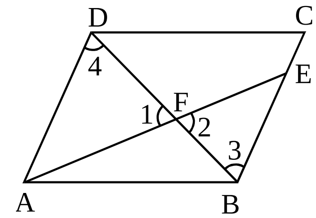
$\therefore\ \Delta ACP \sim \Delta BCQ$.

Ex.41 The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Prove that $DF \times EF = FB \times FA$.

Sol. In ΔAFD and ΔBFE, we have

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [Alternate angles]



So, by AA-criterion of similarity, we have

$\Delta FBE \sim \Delta FDA$

$\Rightarrow \dfrac{FB}{FD} = \dfrac{FD}{FA} \qquad \Rightarrow \dfrac{FB}{DF} = \dfrac{EF}{FA}$

$\Rightarrow$ $DF \times EF = FB \times FA$

Ex.43 In figure, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

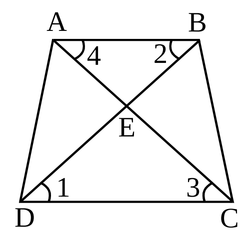
Sol. In ΔEDC and ΔEBA, we have

$\angle 1 = \angle 2$ [Alternate angles]

$\angle 3 = \angle 4$ [Alternate angles]

and, $\angle CED = \angle AEB$[Vertically opposite angles]

$\therefore\ \Delta EDC \sim \Delta EBA$



$\Rightarrow \dfrac{ED}{EB} = \dfrac{EC}{EA}$

$\Rightarrow \dfrac{ED}{EC} = \dfrac{EB}{EA}$ ....(i)

It is given that $\Delta AED \sim \Delta BEC$

$\therefore \dfrac{ED}{EC} = \dfrac{EA}{EB} = \dfrac{AD}{BC}$ ....(ii)

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB}$$

$\Rightarrow (EB)^2 = (EA)^2$

$\Rightarrow \quad EB = EA$

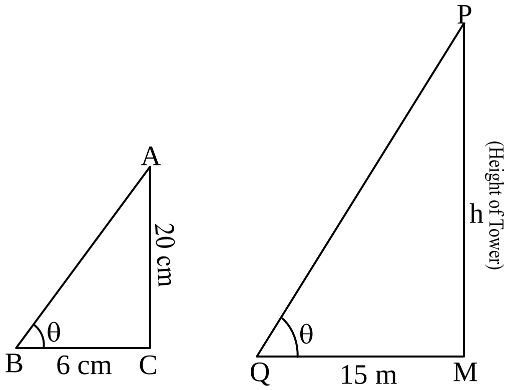
Substituting EB = EA in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \Rightarrow \frac{AD}{BC} = 1$$

$\Rightarrow AD = BC$

Ex.44 A vertical stick 20 cm long casts a shadow 6 cm long on the ground. At the same time, a tower casts a shadow 15 m long on the ground. Find the height of the tower.

Sol. Let the sun's altitude at that moment be θ.



$\Delta PQM \sim \Delta ABC$

$$\Rightarrow \quad \frac{MP}{MQ} = \frac{AC}{CB}$$

$$\Rightarrow \quad \frac{h}{15} = \frac{20}{6}$$

$\therefore$ Height of the tower = 50 m.

Ex.45 If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse then prove that the triangle on each side of the perpendicular are similar to each other and to the original triangle. Also, prove that the square of the perpendicular is equal to the product of the lengths of the two parts of the hypotenuse. **[NCERT]**

Sol. **Given :** A right triangle ABC right angled at B, BD $\perp$ AC.

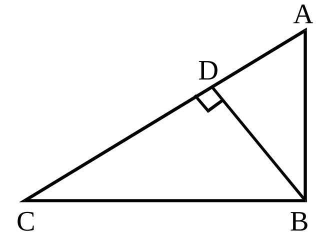
To Prove :

(i) $\Delta ADB \sim \Delta BDC$ (ii) $\Delta ADB \sim \Delta ABC$

(iii) $\Delta BDC \sim \Delta ABC$ (iv) $BD^2 = AD \times DC$

(v) $AB^2 = AD \times AC$ (vi) $BC^2 = CD \times AC$

Proof :



(i) We have,

$\angle ABD + \angle DBC = 90^\circ$

Also, $\angle C + \angle DBC + \angle BDC = 180^\circ$

$\Rightarrow \quad \angle C + \angle DBC + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle C + \angle DBC = 90^\circ$

But, $\angle ABD + \angle DBC = 90^\circ$

$\therefore \quad \angle ABD + \angle DBC = \angle C + \angle DBC$

$\Rightarrow \quad \angle ABD = \angle C$ ....(i)

Thus, in ΔADB and ΔBDC, we have

$\angle ABD = \angle C$ [From (i)]

and, $\angle ADB = \angle BDC$
[Each equal to 90°]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta BDC$

(ii) In ΔADB and ΔABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

(iii) In ΔBDC and ΔABC, we have

$\angle BDC = \angle ABC$
[Each equal to 90°]

$\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

(iv) From (i), we have

$\Delta ADB \sim \Delta BDC$

$$\Rightarrow \quad \frac{AD}{BD} = \frac{BD}{DC} \Rightarrow BD^2 = AD \times DC$$

(v) From (ii), we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \dfrac{AD}{AB} = \dfrac{AB}{AC} \Rightarrow AB^2 = AD \times AC$

(vi) From (iii), we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \dfrac{BC}{AC} = \dfrac{DC}{BC}$

$\Rightarrow BC^2 = CD \times AC$

Ex.46 Prove that the line segments joining the mid points of the sides of a triangle form four triangles, each of which is similar to the original triangle.

Sol. **Given :** ΔABC in which D, E, F are the mid-points of sides BC, CA and AB respectively.

To Prove : Each of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

Proof : Consider triangles AFE and ABC.

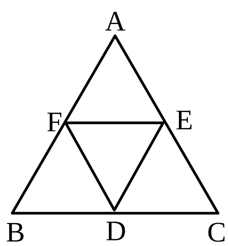
Since F and E are mid-points of AB and AC respectively.

$\therefore$ FE || BC

$\Rightarrow \angle AEF = \angle B$

[Corresponding angles]

Thus, in ΔAFE and ΔABC, we have



$\angle AFE = \angle B$

and, $\angle A = \angle A$ [Common]

$\therefore \Delta AFE \sim \Delta ABC$.

Similarly, we have

$\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall show that $\Delta DEF \sim \Delta ABC$.

Clearly, ED || AF and DE || EA.

$\therefore$ AFDE is a parallelogram.

$\Rightarrow \angle EDF = \angle A$

[$\because$ Opposite angles of a parallelogram are equal]

Similarly, BDEF is a parallelogram.

$\therefore \angle DEF = \angle B$

[$\because$ Opposite angles of a parallelogram are equal]

Thus, in triangles DEF and ABC, we have

$\angle EDF = \angle A$ and $\angle DEF = \angle B$

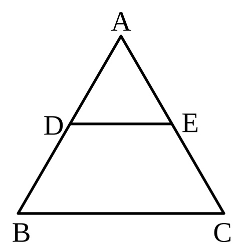
So, by AA-criterion of similarity, we have

$\Delta DEF \sim \Delta ABC$.

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

Ex.47 In ΔABC, DE is parallel to base BC, with D on AB and E on AC. If $\dfrac{AD}{DB} = \dfrac{2}{3}$, find $\dfrac{BC}{DE}$.

Sol. In ΔABC, we have



DE || BC $\Rightarrow \dfrac{AB}{AD} = \dfrac{AC}{AE}$

Thus, in triangles ABC and ADE, we have

$\dfrac{AB}{AD} = \dfrac{AC}{AE}$ and, $\angle A = \angle A$

Therefore, by SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta ADE$

$\Rightarrow \dfrac{AD}{AD} = \dfrac{BC}{DE}$ ....(i)

It is given that

$\dfrac{AD}{DB} = \dfrac{2}{3}$

$\Rightarrow \dfrac{DB}{AD} = \dfrac{3}{2}$

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{3}{2} + 1$$

$$\Rightarrow \frac{DB + AD}{AD} = \frac{5}{2}$$

$$\Rightarrow \frac{AB}{DE} = \frac{5}{2} \qquad ....(ii)$$

From (i) and (ii), we get

$$\frac{BC}{DE} = \frac{5}{2}$$

➤ MORE ON CHARACTERISTIC PROPERTIES

Theorem 1 :

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding medians.

Given : Two triangles ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, AP and DQ are their medians.

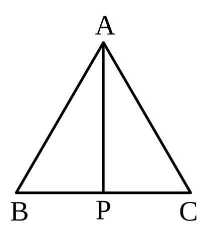


Figure (i)

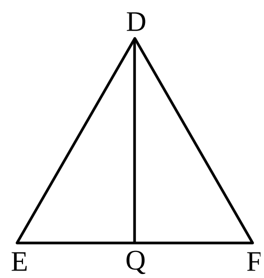


Figure (ii)

To Prove : $\frac{BC}{EF} = \frac{AP}{DQ}$

Proof : Since equiangular triangles are similar.

$\therefore\ \Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{DE} = \frac{2BP}{2EQ}$$

$$\left[\begin{array}{l} \because \text{P and Q are mid-points of BC} \\ \qquad\qquad \text{and EF respectively} \\ \therefore BC = 2BP \text{ and } EF = 2EQ \end{array}\right]$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} \qquad ....(ii)$$

Now, in ΔABP and ΔDFQ, we have

$$\frac{AB}{DE} = \frac{BP}{EQ} \qquad \text{[From (ii)]}$$

and, $\angle B = \angle E$ [Given]

So, by SAS-criterion of similarity, we have

$\Delta ABP \sim \Delta DEQ$

$$\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ} \qquad ....(iii)$$

From (i) and (iii), we get

$$\frac{BC}{EF} = \frac{AP}{DQ}$$

Hence, the ratio of the corresponding sides is same as the ratio of corresponding medians.

Theorem 5 :

If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.

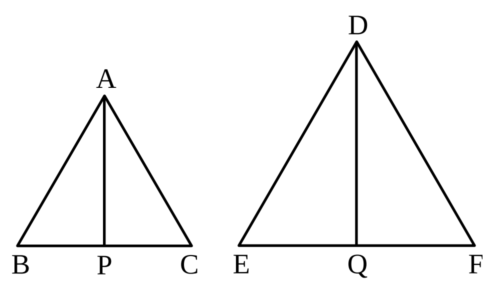
Given : ΔABC and ΔDEF in which AP and DQ are the medians such that **[NCERT]**

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$$

To Prove : $\Delta ABC \sim \Delta DEF$

Proof : We have,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$$



$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{AP}{DQ}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ}$$

$\Rightarrow \Delta ABP \sim \Delta DEQ$ [By SSS-similarity]

$\Rightarrow \angle B = \angle E$

Now, in ΔABC and ΔDEF, we have

$$\frac{AB}{DE} = \frac{BC}{EF} \qquad \text{[Given]}$$

and, $\angle B = \angle E$

So, by SAS-criterion of similarity, we get

$\Delta ABC \sim \Delta DEF$

Theorem 6 :

If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then the two triangles are similar.

[NCERT]

Given : Two triangle ABC and DEF in which AP and DQ are the medians such that

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}.$$

To Prove : $\Delta ABC \sim \Delta DEF$

Construction : Produce AP to G so that PG = AP. Join CG. Also, produce DQ to H so that QH = DQ. Join FH.

Proof : In ΔAPB and ΔGPC, we have

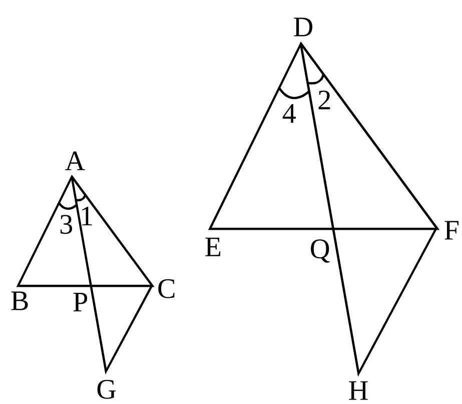
BP = CP [$\because$ AP is the median]
AP = GP [By construction]

and, $\angle APB = \angle CPG$ [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

$\Delta APB \cong \Delta GPC$

$\Rightarrow$ AG = GC ....(i)



Again, In ΔDQE and ΔHQF, we have

EQ = FQ [$\because$ DQ is the median]

DQ = HQ [By construction]

and, $\angle DQE = \angle HQF$ [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

$\Delta DQE \cong \Delta HQF$

$\Rightarrow$ DE = HF ....(ii)

Now, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$ [Given]

$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AP}{DQ}$$

[$\because$ AB = GC and DE = HF (from (i) and (ii))]

$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{2AP}{2DQ}$$

$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AG}{DH}$$

[$\because$ 2AP = AG and 2DQ = DH]

$\Rightarrow \Delta AGC \sim \Delta DHF$

[By SSS-criterion of similarity]

$\Rightarrow \angle 1 = \angle 2$

Similarly, we have

$\angle 3 = \angle 4$

$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle A = \angle D$ ....(iii)

Thus, in ΔABC and ΔDEF, we have

$\angle A = \angle D$ [From (iii)]

and, $\frac{AB}{DE} = \frac{AC}{DF}$ [Given]

So, by SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta DEF$

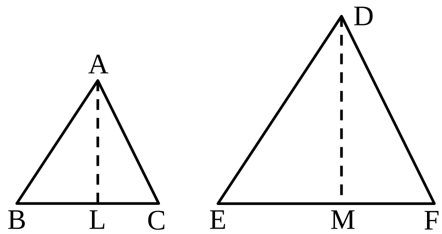
➤ AREAS OF TWO SIMILAR TRIANGLES

Theorem 1 :

The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

Given : Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$.

To Prove : $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$



Construction : Draw AL $\perp$ BC and DM $\perp$ EF.

Proof : Since similar triangles are equiangular and their corresponding sides are proportional. Therefore,

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ....(i)

Thus, in ΔALB and ΔDME, we have

$\Rightarrow \angle ALB = \angle DME$ [Each equal to 90º]

and, $\angle B = \angle E$ [From (i)]

So, by AA-criterion of similarity, we have

$\Delta ALB \sim \Delta DME$

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$ ....(ii)

From (i) and (ii), we get

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM}$ ....(iii)

Now,

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(EF \times DM)}$$

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} \left[\text{From (iii)}, \frac{BC}{EF} = \frac{AL}{DM}\right]$$

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC^2}{EF^2}$$

But, $\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF}$

$$\Rightarrow \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

Hence, $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Theorem 2 :

If the areas of two similar triangles are equal, then the triangles are congruent i.e. equal and similar triangles are congruent.

Given : Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$ and Area (ΔABC) = Area (ΔDEF).

To Prove : We have,

$\Delta ABC \cong \Delta DEF$

Proof : $\Delta ABC \sim \Delta DEF$

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

In order to prove that $\Delta ABC \cong \Delta DEF$, it is sufficient to show that AB = DE, BC = EF and AC = DF.

Now, Area (ΔABC) = Area (ΔDEF)

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = 1$$

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$$

$$\left[\because \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}\right]$$

$\Rightarrow AB^2 = DE^2, BC^2 = EF^2$ and $AC^2 = DF^2$

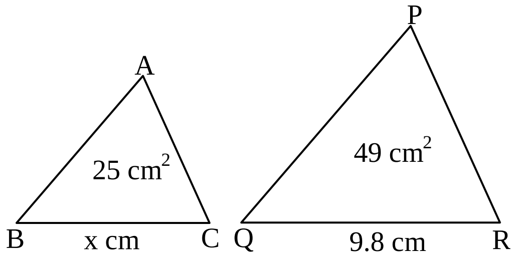
$\Rightarrow$ AB = DE, BC = EF and AC = DF

Hence, $\Delta ABC \cong \Delta DEF$.

❖ EXAMPLES ❖

Ex.48 The areas of two similar triangles ΔABC and ΔPQR are 25 cm^2 and 49 cm^2 respectively. If QR = 9.8 cm, find BC.

Sol. It is being given that $\Delta ABC \sim \Delta PQR$, ar ($\Delta ABC$) = 25 cm^2 and ar (ΔPQR) = 49 cm^2. We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{25}{49} = \frac{x^2}{(9.8)^2}, \text{ where } BC = x \text{ cm}$$

$$\Rightarrow \quad x^2 = \left(\frac{25}{49} \times 9.8 \times 9.8\right)$$

$$\Rightarrow x = \sqrt{\frac{25}{49} \times 9.8 \times 9.8} = \left(\frac{5}{7} \times 9.8\right) = (5 \times 1.4) = 7.$$

Hence BC = 7 cm.

Ex.49 In two similar triangles ABC and PQR, if their corresponding altitudes AD and PS are in the ratio 4 : 9, find the ratio of the areas of ΔABC and ΔPQR.

Sol. Since the areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

$$\therefore \quad \frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \quad \frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

[$\because$ AD : PS = 4 : 9]

Hence, Area (ΔABC) : Area (ΔPQR) = 16 : 81

Ex.50 If ΔABC is similar to ΔDEF such that $\Delta DEF = 64 \text{ cm}^2$, DE = 5.1 cm and area of $\Delta ABC = 9 \text{ cm}^2$. Determine the area of AB.

Sol. Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$$\therefore \quad \frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \quad \frac{9}{64} = \frac{AB^2}{(5.1)^2}$$

$$\Rightarrow \quad AB = \sqrt{3.65} \Rightarrow AB = 1.912 \text{ cm}$$

Ex.51 If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is 16cm^2 and the area of ΔDEF is 25cm^2 and BC = 2.3 cm. Find the length of EF.

Sol. We have,

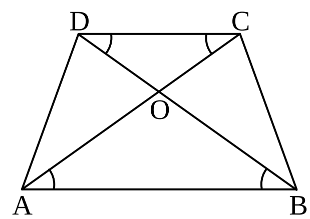
$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{16}{25} = \frac{(2.3)^2}{EF^2} \Rightarrow EF = \sqrt{8.265}$$

$$= 2.875 \text{ cm}$$

Ex.52 In a trapezium ABCD, O is the point of intersection of AC and BD, AB || CD and AB = 2 × CD. If the area of $\Delta AOB = 84 \text{ cm}^2$. Find the area of ΔCOD.

Sol. In ΔAOB and ΔCOD, we have



$\angle OAB = \angle OCD$ (alt. int. $\angle$s)

$\angle OBA = \angle ODC$ (alt. int. $\angle$s)

$\therefore \quad \Delta AOB \sim \Delta COD$ [By AA-similarity]

$$\Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2}$$

[$\because$ AB = 2 × CD]

$$= \frac{4 \times CD^2}{CD^2} = 4$$

$\Rightarrow$ ar (ΔCOD) = 1/4 × ar (ΔAOB)

$$= \left(\frac{1}{4} \times 84\right) \text{cm}^2 = 21 \text{cm}^2$$

Hence, the area of ΔCOD is 21 cm^2.

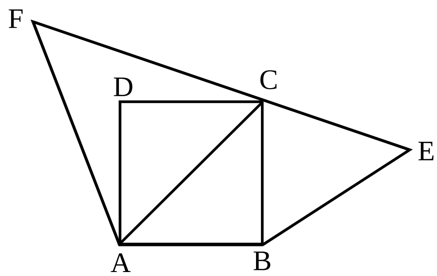
Ex.53 Prove that the area of the triangle BCE described on one side BC of a square ABCD as base is one half the area of the similar triangle ACF described on the diagonal AC as base.

Sol. ABCD is a square. ΔBCE is described on side BC is similar to ΔACF described on diagonal AC.

Since ABCD is a square. Therefore,

AB = BC = CD = DA and, AC = $\sqrt{2}$ BC

[$\because$ Diagonal = $\sqrt{2}$ (Side)]



Now, $\Delta BCE \sim \Delta ACF$

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$$

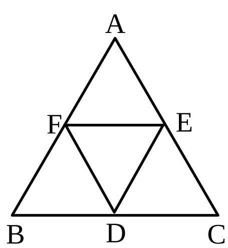
$$\Rightarrow \text{Area }(\Delta BCE) = \frac{1}{2}\text{ Area }(\Delta ACF)$$

Ex.54 D, E, F are the mid-point of the sides BC, CA and AB respectively of a ΔABC. Determine the ratio of the areas of ΔDEF and ΔABC.

Sol. Since D and E are the mid-points of the sides BC and AB respectively of ΔABC. Therefore,

DE $\parallel$ BA

$\Rightarrow$ DE $\parallel$ FA ....(i)



Since D and F are mid-points of the sides BC and AB respectively of ΔABC. Therefore,

DF $\parallel$ CA $\Rightarrow$ DF $\parallel$ AE

From (i), and (ii), we conclude that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in ΔDEF and ΔABC, we have

$\angle FDE = \angle A$

[Opposite angles of parallelogram AFDE]

and, $\angle DEF = \angle B$

[Opposite angles of parallelogram BDEF]

So, by AA-similarity criterion, we have

$\Delta DEF \sim \Delta ABC$

$$\Rightarrow \frac{\text{Area }(\Delta DEF)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{(1/2AB)^2}{AB^2} = \frac{1}{4}$$

$$\left[\because DE = \frac{1}{2}AB\right]$$

Hence, Area (ΔDEF) : Area (ΔABC) = 1 : 4.

Ex.55 D and E are points on the sides AB and AC respectively of a ΔABC such that DE $\parallel$ BC and divides ΔABC into two parts, equal in area. Find $\frac{BD}{AB}$.

Sol. We have,

Area (ΔADE) = Area (trapezium BCED)

$\Rightarrow$ Area (ΔADE) + Area (ΔADE)

= Area (trapezium BCED) + Area (ΔADE)

$\Rightarrow$ 2 Area (ΔADE) = Area (ΔABC)

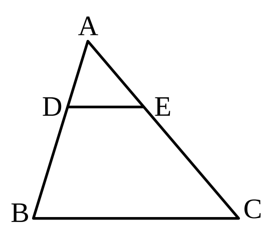
In ΔADE and ΔABC, we have

$\angle ADE = \angle B$

[$\because$ DE $\parallel$ BC $\therefore$ $\angle ADE = \angle B$ (Corresponding angles)]

and, $\angle A = \angle A$ [Common]

$\therefore$ $\Delta ADE \sim \Delta ABC$



$$\Rightarrow \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta ADE)}{2\text{ Area }(\Delta ADE)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{AD}{AB}\right)^2 \Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow AB = \sqrt{2}\,AD \Rightarrow AB = \sqrt{2}\,(AB - BD)$$

$$\Rightarrow (\sqrt{2} - 1)\,AB = \sqrt{2}\,BD$$

$$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

Ex.56 Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

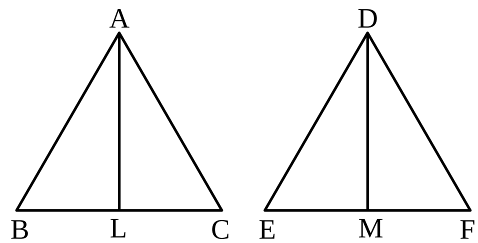
Sol. Let ΔABC and ΔDEF be the given triangles such that AB = AC and DE = DF, $\angle A = \angle D$.

and, $\dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \dfrac{16}{25}$ ....(i)

Draw $AL \perp BC$ and $DM \perp EF$.

Now, AB = AC, DE = DF

$\Rightarrow \dfrac{AB}{AC} = 1$ and $\dfrac{DE}{DF} = 1$



$\Rightarrow \dfrac{AB}{AC} = \dfrac{DE}{DF} \quad \Rightarrow \dfrac{AB}{DE} = \dfrac{AC}{DF}$

Thus, in triangles ABC and DEF, we have

$\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$ [Given]

So, by SAS-similarity criterion, we have

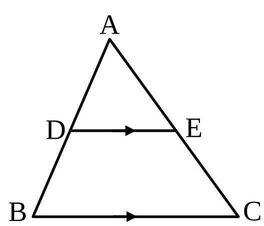
$\Delta ABC \sim \Delta DEF$

$\Rightarrow \dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \dfrac{AL^2}{DM^2}$

$\Rightarrow \dfrac{16}{25} = \dfrac{AL^2}{DM^2}$ [Using (i)]

$\Rightarrow \dfrac{AL}{DM} = \dfrac{4}{5} \Rightarrow AL : DM = 4 : 5$

Ex.57 In the given figure, DE || BC and DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.



Sol. $\Delta ADE \sim \Delta ABC$.

$\therefore \dfrac{ar(\Delta ADE)}{ar(\Delta ABC)} = \dfrac{DE^2}{BC^2} = \left(\dfrac{DE}{BC}\right)^2 = \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$

Let ar (ΔADE) = 9x sq units

Then, ar (ΔABC) = 25x sq units

ar (trap. BCED) = ar (ΔABC) – ar (ΔADE)

= (25x – 9x) = (16x) sq units

$\therefore \dfrac{ar(\Delta ADE)}{ar(\text{trap.}BCED)} = \dfrac{9x}{16x} = \dfrac{9}{16}$

➢ PYTHAGORAS THEOREM

Theorem 1 :

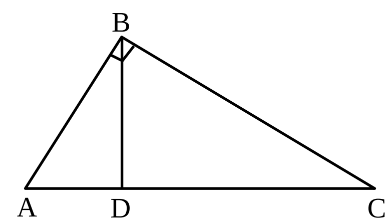
In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : A right-angled triangle ABC in which $\angle B = 90^\circ$.

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$.

i.e., $AC^2 = AB^2 + BC^2$

Construction : From B draw $BD \perp AC$.



Proof : In triangle ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90º]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \dfrac{AD}{AB} = \dfrac{AB}{AC}$

$\left[\because \begin{array}{l}\text{In similar triangles corresponding} \\ \text{sides are proportional}\end{array}\right]$

$\Rightarrow AB^2 = AD \times AC$ ....(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90º]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \dfrac{DC}{BC} = \dfrac{BC}{AC}$

$\left[\because \begin{array}{l}\text{In similar triangles corresponding} \\ \text{sides are proportional}\end{array}\right]$

$\Rightarrow$ $BC^2 = AC \times DC$ ....(ii)

Adding equation (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow$ $AB^2 + BC^2 = AC\ (AD + DC)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC^2$

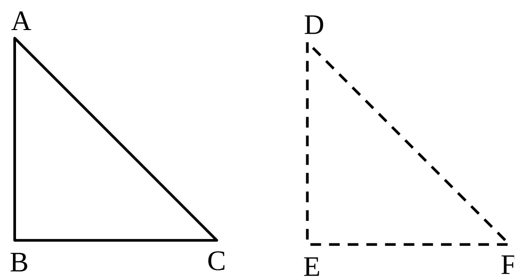
Hence, $AC^2 = AB^2 + BC^2$

The converse of the above theorem is also true as proved below.

Theorem 2 : (Converse of Pythagoras Theorem).

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the side is a right angle.

Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$



Construction : Construct a triangle DEF such that DE = AB, EF = BC and $\angle E = 90^o$,

Proof : In order to prove that $\angle B = 90^o$, it is sufficient to show that $\Delta ABC \sim \Delta DEF$.

For this we proceed as follows :

Since ΔDEF is a right angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have

$DF^2 = DE^2 + EF^2$

$\Rightarrow$ $DF^2 = AB^2 + BC^2$

[$\because$ DE = AB and EF = BC

(By construction)]

$\Rightarrow$ $DF^2 = AC^2$ [$\because AB^2 + BC^2 = AC^2$ (Given)]

$\Rightarrow$ DF = AC ....(i)

Thus, in ΔABC and ΔDEF, we have

AB = DE, BC = EF [By construction]

and, AC = DF [From equation (i)]

$\therefore$ $\Delta ABC \cong \Delta DEF$

$\Rightarrow$ $\angle B = \angle E = 90^o$

Hence, ΔABC is a right triangle right angled at B.

❖ EXAMPLES ❖

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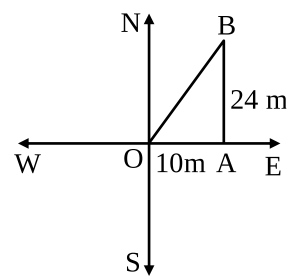
Ex.59 A man goes 10 m due east and then 24 m due north. Find the distance from the starting point.

Sol. Let the initial position of the man be O and his final position be B. Since the man goes 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right-angled at A such that OA = 10 m and AB = 24 m.



By Phythagoras theorem, we have

$OB^2 = OA^2 + AB^2$

$\Rightarrow$ $OB^2 = 10^2 + 24^2 = 100 + 576 = 676$

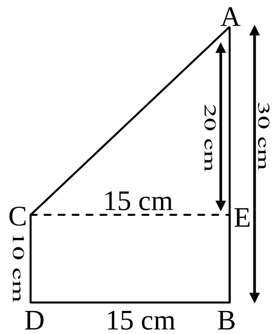
$\Rightarrow$ $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

Ex.60 Two towers of heights 10 m and 30 m stand on a plane ground. If the distance between their feet is 15 m, find the distance between their tops.

Sol. $AC^2 = (15)^2 + (20)^2 = 625$

$\Rightarrow$ AC = 25 m.



Ex.61 In Fig., ΔABC is an obtuse triangle, obtuse angled at B. If $AD \perp CB$, prove that

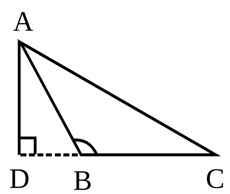
$AC^2 = AB^2 + BC^2 + 2BC \times BD$

Sol. **Given :** An obtuse triangle ABC, obtuse-angled at B and AD is perpendicular to CB produced.

To Prove : $AC^2 = AB^2 + BC^2 + 2BC \times BD$

Proof : Since ΔADB is a right triangle right angled at D. Therefore, by Pythagoras theorem, we have

$AB^2 = AD^2 + DB^2$ ....(i)



Again ΔADC is a right triangle right angled at D.

Therefore, by Phythagoras theorem, we have

$AC^2 = AD^2 + DC^2$

$\Rightarrow$ $AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow$ $AC^2 = AD^2 + DB^2 + BC^2 + 2BC \cdot BD$

$\Rightarrow$ $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

[Using (i)]

Hence, $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

Ex.62 In figure, $\angle B$ of ΔABC is an acute angle and $AD \perp BC$, prove that

$AC^2 = AB^2 + BC^2 - 2BC \times BD$

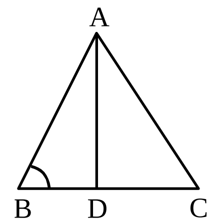
Sol. **Given :** A ΔABC in which $\angle B$ is an acute angle and $AD \perp BC$.

To Prove : $AC^2 = AB^2 + BC^2 - 2BC \times BD$.

Proof : Since ΔADB is a right triangle right-angled at D. So, by Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2$ ....(i)

Again ΔADC is a right triangle right angled at D.



So, by Pythagoras theorem, we have

$AC^2 = AD^2 + DC^2$

$\Rightarrow$ $AC^2 = AD^2 + (BC - BD)^2$

$\Rightarrow$ $AC^2 = AD^2 + (BC^2 + BD^2 - 2BC \cdot BD)$

$\Rightarrow$ $AC^2 = (AD^2 + BD^2) + BC^2 - 2BC \cdot BD$

$\Rightarrow$ $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$

[Using (i)]

Hence, $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$

Ex.65 ABCD is a rhombus. Prove that

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

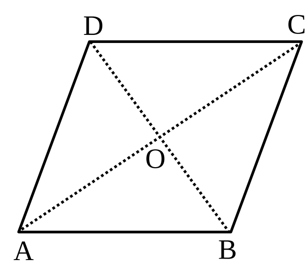
Sol. Let the diagonals AC and BD of rhombus ABCD intersect at O.

Since the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$

and AO = CO, BO = OD.

Since ΔAOB is a right triangle right-angle at O.



$\therefore$ $AB^2 = OA^2 + OB^2$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 \quad \left[\begin{array}{l}\because OA = OC \\ \text{and } OB = OD\end{array}\right]$$

$\Rightarrow$ $4AB^2 = AC^2 + BD^2$ ....(i)

Similarly, we have

$4BC^2 = AC^2 + BD^2$ ....(ii)

$4CD^2 = AC^2 + BD^2$ ....(iii)

and, $4AD^2 = AC^2 + BD^2$ ....(iv)

Adding all these results, we get

$4(AB^2 + BC^2 + AD^2) = 4(AC^2 + BD^2)$

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Ex.66 P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that :

(i) $4AQ^2 = 4AC^2 + BC^2$

(ii) $4BP^2 = 4BC^2 + AC^2$

(iii) $(4AQ^2 + BP^2) = 5AB^2$

Sol. (i) Since ΔAQC is a right triangle right-angled at C.

$\therefore \quad AQ^2 = AC^2 + QC^2$

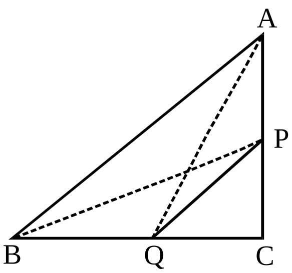
$\Rightarrow \quad 4AQ^2 = 4AC^2 + 4QC^2$

[Multiplying both sides by 4]

$\Rightarrow \quad 4AQ^2 = 4AC^2 + (2QC)^2$

$\Rightarrow \quad 4AQ^2 = 4AC^2 + BC^2 \quad [\because \ BC = 2QC]$

(ii) Since ΔBPC is a right triangle right-angled at C.



$\therefore \quad BP^2 = BC^2 + CP^2$

$\Rightarrow \quad 4BP^2 = 4BC^2 + 4CP^2$

[Multiplying both sides by 4]

$\Rightarrow \quad 4BP^2 = 4BC^2 + (2CP)^2$

$\Rightarrow \quad 4BP^2 = 4BC^2 + AC^2 \quad [\because \ AC = 2CP]$

(iii) From (i) and (ii), we have

$4AQ^2 = 4AC^2 + BC^2$ and, $4BC^2 = 4BC^2 + AC^2$

$\therefore 4AQ^2 + 4BP^2 = (4AC^2 + BC^2) + (4BC^2 + AC^2)$

$\Rightarrow 4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$

$\Rightarrow 4(AQ^2 + BP^2) = 5AB^2$

[In ΔABC, we have $AB^2 = AC^2 + BC^2$]

Ex.67 From a point O in the interior of a ΔABC, perpendicular OD, OE and OF are drawn to the sides BC, CA and AB respectively. Prove that : **[NCERT]**

(i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Sol. Let O be a point in the interior of ΔABC and let $OD \perp BC$, $OE \perp CA$ and $OF \perp AB$.

(i) In right triangles ΔOFA, ΔODB and ΔOEC, we have

$OA^2 = AF^2 + OF^2$

$OB^2 = BD^2 + OD^2$

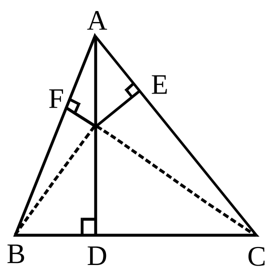
and, $OC^2 = CE^2 + OE^2$

Adding all these results, we get

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow \quad AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

(ii) In right triangles ΔODB and ΔODC, we have



$OB^2 = OD^2 + BD^2$

and, $OC^2 = OD^2 + CD^2$

$\therefore OB^2 - OC^2 = (OD^2 + BD^2) - (OD^2 + CD^2)$

$\Rightarrow \quad OB^2 - OC^2 = BD^2 - CD^2 \quad$ ....(i)

Similarity, we have

$OC^2 - OA^2 = CE^2 - AE^2 \quad$ ....(ii)

and, $OA^2 - OB^2 = AF^2 - BF^2 \quad$ ....(iii)

Adding (i), (ii) and (iii), we get

$(OB^2 - OC^2) + (OC^2 - OA^2) + (OA^2 - OB^2)$

$= (BD^2 - CD^2) + (CE^2 - AE^2) + (AF^2 - BF^2)$

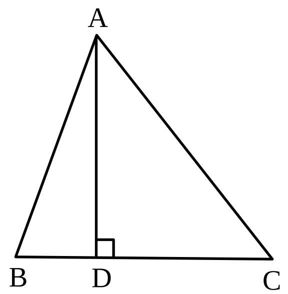
$\Rightarrow (BD^2 + CE^2 + AF^2) - (AE^2 + CD^2 + BF^2) = 0$

$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$

Ex.69 In a ΔABC, $AD \perp BC$ and $AD^2 = BC \times CD$. Prove that ΔABC is a right triangle.

Sol. In right triangles ADB and ADC, we have

$$AB^2 = AD^2 + BD^2 \quad ....(i)$$



and, $AC^2 = AD^2 + DC^2 \quad ....(ii)$

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2\,AD^2 \times BD^2 + DC^2$

$\Rightarrow \; AB^2 + AC^2 = 2BD \times CD + BD^2 + DC^2$

$[\because AD^2 = BD \times CD \text{ (Given)}]$

$\Rightarrow \; AB^2 + AC^2 = (BD + CD)^2 = BC^2$

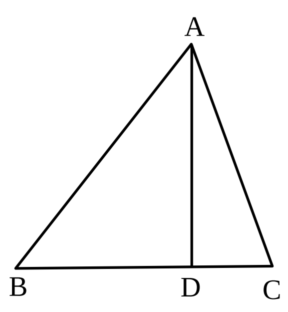
Thus, in Δ ABC, we have

$AB^2 = AC^2 + BC^2$

Hence, Δ ABC, is a right triangle right-angled at A.

Ex.70 The perpendicular AD on the base BC of a ΔABC intersects BC at D so that DB = 3 CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

Sol. We have,



DB = 3CD

$\therefore \; BC = BD + DC$

$\Rightarrow \; BC = 3\,CD + CD$

$\Rightarrow \; BD = 4\,CD \Rightarrow CD = \frac{1}{4}BC$

$\therefore \; CD = \frac{1}{4}BC$ and $BD = 3CD = \frac{1}{4}BC \quad ....(i)$

Since ΔABD is a right triangle right-angled at D.

$\therefore \; AB^2 = AD^2 + BD^2 \quad ....(ii)$

Similarly, ΔACD is a right triangle right angled at D.

$\therefore \; AC^2 = AD^2 + CD^2 \quad ....(iii)$

Subtracting equation (iii) from equation (ii) we get

$AB^2 - AC^2 = BD^2 - CD^2$

$\Rightarrow \; AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$

$\left[\text{From (i) } CD = \frac{1}{4}BC,\ BD = \frac{3}{4}BC\right]$

$\Rightarrow \; AB^2 - AC^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2$

$\Rightarrow \; AB^2 - AC^2 = \frac{1}{2}BC^2$

$\Rightarrow \; 2(AB^2 - AC^2) = BC^2$

$\Rightarrow \; 2AB^2 = 2AC^2 + BC^2$

IMPORTANT POINTS TO BE REMEMBERED

1. Two figures having the same shape but not necessarily the same size are called similar figures.

2. All congruent figures are similar but the converse is not true.

3. Two polygones having the same number of sides are similar, if

   (a) Their corresponding angles are equal and

   (b) Their corresponding sides are proportional

   (i.e., in the same ratio)

4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side of the triangle.

6. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

7. If a line through one vertex of a triangle divides the opposite side in the ratio of other two sides, then the line bisects the angle at the vertex.

8. The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

9. The line drawn from the mid-point of two sides of a triangle is parallel of another side bisects the third side.

10. The line joining the mid-points of two sides of a triangle is parallel to the third side.

11. The diagonals of a trapezium divide each other proportionally.

12. If a diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.

13. Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

14. If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.

15. **AAA similarity criterion :** If in two triangles, corresponding angles are equal, then the triangles are similar.

16. **AA Similarity criterion :** If in two triangles, two angles of one triangle are respectively equal the two angles of the other triangle, then the two triangles are similar.

17. **SSS Similarity criterion :** If in two triangles, corresponding sides are in the same ratio, then the two triangles are similar.

18. If one angle of a triangles is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar.

19. If two triangles are equiangular, then

    (i) The ratio of the corresponding sides is same as the ratio of corresponding median.

    (ii) The ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.

    (iii) The ratio of the corresponding sides is same as the ratio of the corresponding altitudes.

20. If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite side in the same ratio, then the triangles are similar.

21. If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.

22. If two sides and a median bisecting the third side of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.

23. The ratio of the areas of two similar triangles is equal to the ratio of

    (i) The squares of any two corresponding sides

(ii) The squares of the corresponding altitudes.

(iii) The squares of the corresponding medians.

(iv) The squares of the corresponding angle bisector segments.

24. If the areas of two similar triangles are equal, then the triangles are congruent i.e., equal and similar triangles congruent.

25. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.

26. **Pythagoras Theorem :** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

27. **Converse of Pythagoras Theorem :** If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle.

28. In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.

29. Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

30. Three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.